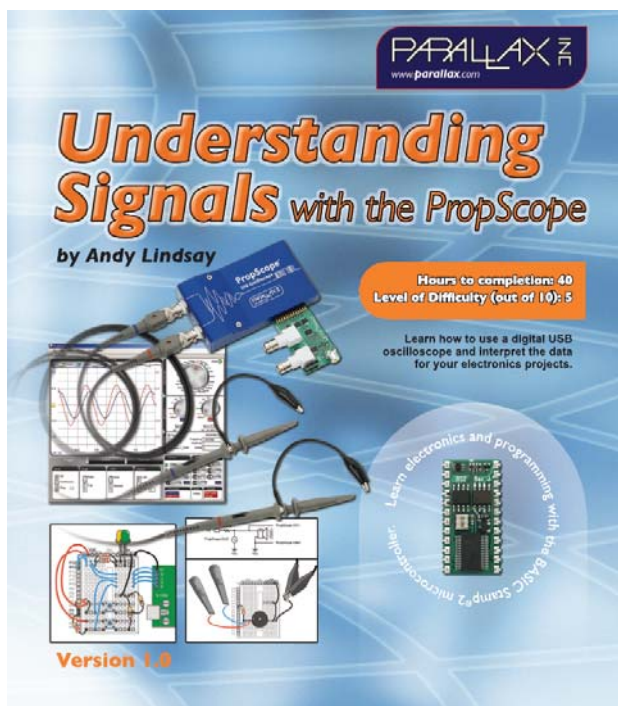


# Understanding Signals with the PropScope Supplement & Errata

This document contains supplementary materials for the Understanding Signals with the PropScope v1.0 text (#122-32225). Errata for the text, if any, will also be included here as well.



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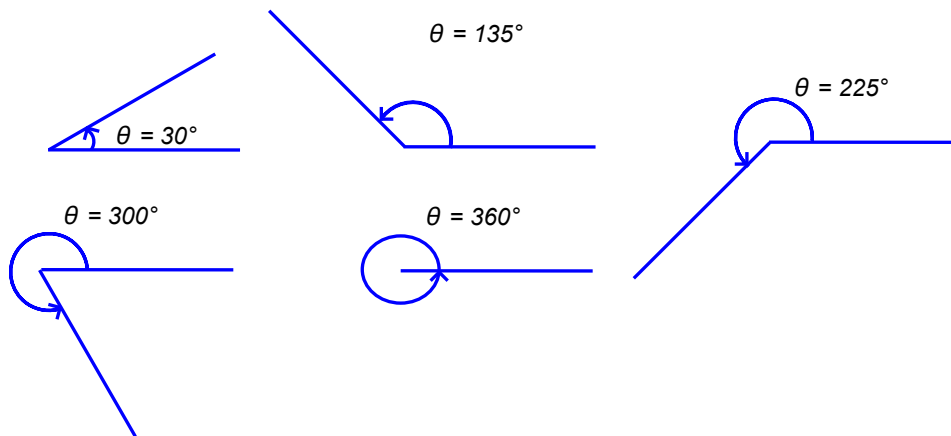
# Sine Wave Math

This supplemental activity introduces the math behind the sine waves you have been measuring in Chapter 7 of *Understanding Signals with the PropScope*. Properties such as amplitude, frequency, offset, and *phase delay* all have corresponding values in an equation called the sine function. In this activity, you will experiment with basic sine function calculations, graphing a sine function, and sine function values for different amplitudes, frequencies, offsets and phase delays.

## DEGREE MEASUREMENTS

One way of measuring the angle between two lines at their intersection is by *degrees*, which is abbreviated with the degree symbol  $^{\circ}$ . The Greek letter theta ( $\theta$ ) is commonly used to represent a value that is an angle measurement. Figure 1 shows some examples of degree measurements, including  $30^{\circ}$ ,  $135^{\circ}$ ,  $225^{\circ}$ ,  $300^{\circ}$ , and  $360^{\circ}$ . Since degrees are the number of  $360^{\text{th}}$ s of a full circle, there's only one line visible in the  $360^{\circ}$  angle because the second line has done a full circle and overlaps with the first.

Figure 1: Angle Measurements



## ANGLES IN RIGHT TRIANGLES AND SIMPLE TRIGONOMETRY CALCULATIONS

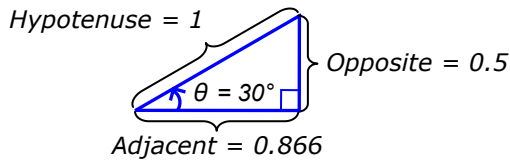
*Sine*, *cosine* and *tangent* are three examples of calculations introduced in trigonometry classes. They are called *trigonometric functions*, and are often referred to as “trig functions” in common speech. The abbreviations for those trig functions are sin, cos, and tan, and they describe the ratios of the sides of a right triangle based on its angles. A right triangle is one with a  $90^{\circ}$  corner. The ratios of the various sides of the right triangle depend on the other two angles. In other words, divide the length of one side by another side, and you’ll get the same result, regardless of the size of the triangle.

Figure 2 shows an example of sin, cos, and tan calculations with a  $30^{\circ}$  right triangle. The labels for each side are the common ones introduced in trigonometry classes. This example triangle has a hypotenuse with a length of 1. It’s a right triangle with a  $30^{\circ}$  angle between the sides labeled adjacent and hypotenuse. As a result of these two properties, the opposite side has to have a length of 0.5 and the adjacent side has to have a length of 0.866.

- ✓ Examine the sin, cos and tan calculations in Figure 2.
- ✓ Try them on the larger  $30^{\circ}$  right triangle.
- ✓ Repeat the calculations for the two different size  $45^{\circ}$  right triangles. The calculations should be different from the  $30^{\circ}$  triangle calculations, but the results should be the same for both sizes of  $45^{\circ}$  triangle.

**Figure 2: Example Trig Calculations**

**30° right triangle example**



$$\sin(\theta) = \text{opposite} \div \text{hypotenuse}$$

$$\sin(30^\circ) = 5 \div 10 = 0.5$$

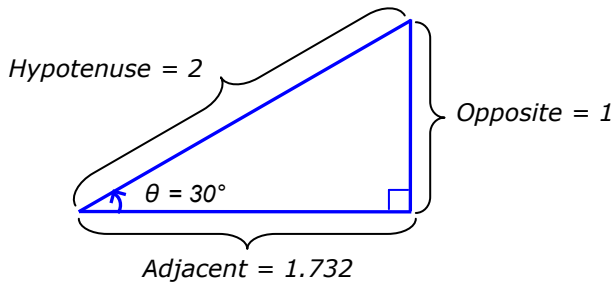
$$\cos(\theta) = \text{adjacent} \div \text{hypotenuse}$$

$$\cos(30^\circ) = 8.66 \div 10 = 0.866$$

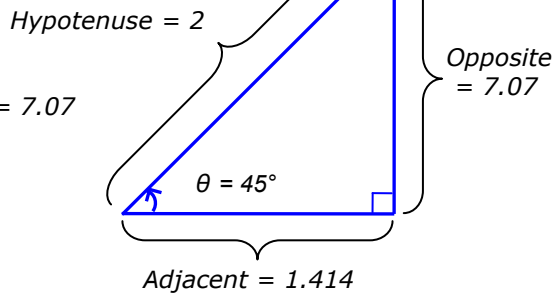
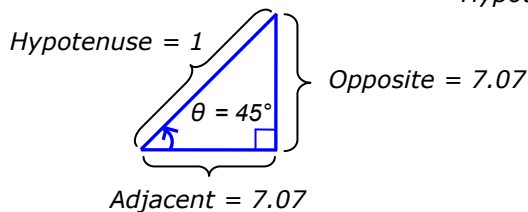
$$\tan(\theta) = \text{opposite} \div \text{adjacent}$$

$$\tan(30^\circ) = 5 \div 8.66 = 0.577$$


**Same angle, different size**



**Different angle, two sizes**



The sine, cosine, and tangent relationships are dictated by the angle between the adjacent and hypotenuse sides of a right triangle. The size of the triangle does not matter, only the angle. That's why the results of the sin, cos, and tan calculations were the same for both sizes of a given triangle with the same angles, and it's also why the calculations differed for triangles with different angles.

	<p><b>The angles in a triangle always add up to 180°</b></p> <p>So, the other corner of the 30° right triangle has to have a 60° angle because <math>30^\circ + 90^\circ + 60^\circ = 180^\circ</math>. Likewise, the other angle in the 45° right triangle is 45° because <math>45^\circ + 90^\circ + 45^\circ = 180^\circ</math></p>
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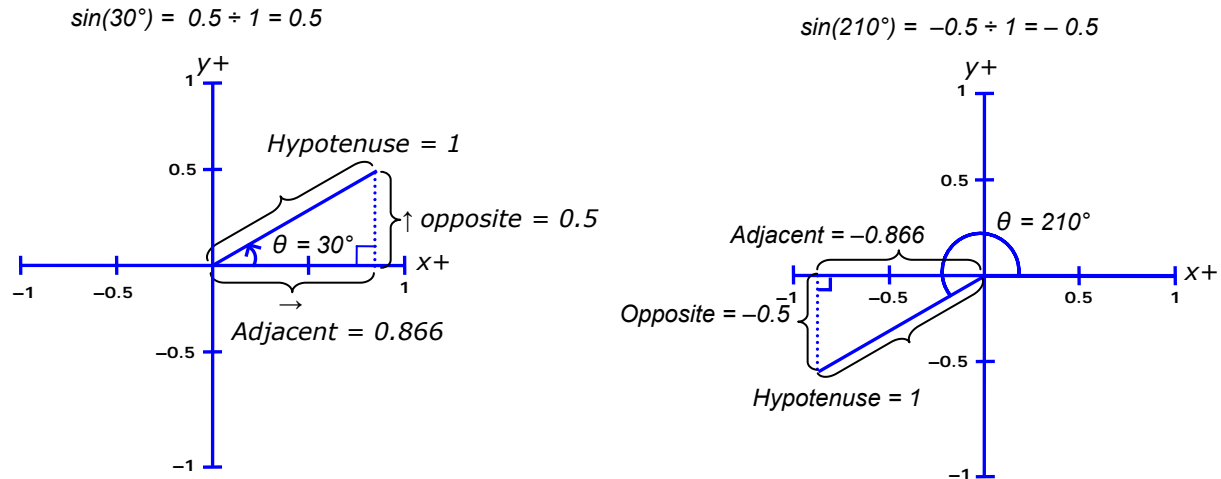
**SINE CALCULATIONS FOR ANGLES FROM 0° TO 360°**

Think of the hypotenuse in Figure 3 as attached to the xy axis origin, the point where the x and y axes meet. Also think of the hypotenuse as able to spin freely about that point. With this arrangement, you can draw a line, either straight up or straight down from the end of the hypotenuse to the x-axis to make a right triangle. Then, its height (opposite side) can be measured by the y-axis, and its width (adjacent side) can be measured with the x-axis.

The 30° calculation looks familiar, but the second example with the 210° has a twist. The adjacent side is a negative x-axis value, and the opposite side is a negative y axis value. We only need the opposite value for the sine calculation, but if you calculate cos, you'd have to take the adjacent side into account. Likewise, if you calculate tan, you'd have to use both the opposite and adjacent sides as well as their signs.

**Figure 3: Example Sine Calculations**

$$\sin(\theta) = \text{opposite} \div \text{hypotenuse}$$

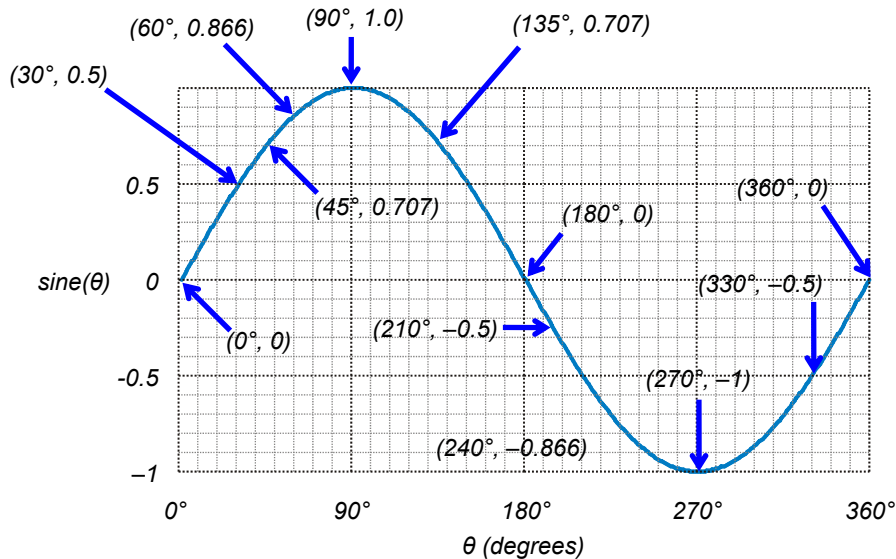


### A SINE WAVE – SINE CALCULATIONS FROM 0 TO 360°

If you calculate the  $\sin(1^\circ)$ , then  $\sin(2^\circ)$ , then  $\sin(3^\circ)$ , and so-on, all the way through  $\sin(360^\circ)$  and plot them all on a graph, it would look similar to Figure 4. In this graph, each successive degree angle  $\theta$  is an x-axis value, and the calculated  $\sin(\theta)$  result is a y-axis value. Up to this point, we have calculated three of the  $\theta$ ,  $\sin(\theta)$  points shown on this graph:  $(30^\circ, 0.5)$ ,  $(45^\circ, 0.707)$ , and  $(210^\circ, -0.5)$ .

- ✓ Locate the three points on this graph:  $(30^\circ, 0.5)$ ,  $(45^\circ, 0.707)$ , and  $(210^\circ, -0.5)$

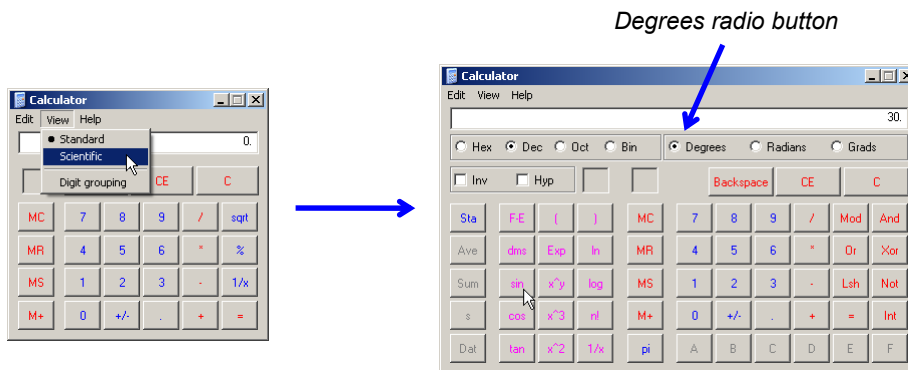
**Figure 4: Sine ( $\theta$ ) vs.  $\theta$  with Example Degree Measurements**



The sine function on your calculator can be used to calculate points on a sine wave. The Windows calculator in Figure 5 has a Scientific mode that includes trigonometric functions like sine, cosine, and tangent (the sin, cos, and tan buttons).


- ✓ Run Windows Calculator by clicking Start → All Programs → Accessories → Calculator.
- ✓ If your calculator looks like the one on the left, click View and select Scientific.
- ✓ Make sure the Degrees radio button is selected.

Figure 5: Windows Calculator



Let's test the calculator's sine function. The sine of  $30^\circ$  is 0.5.

- ✓ Type 30 into the calculator's number field and click the sin button. Verify that the calculator displays the result 0.5.
- ✓ Try it for a few more  $\theta$ ,  $\text{sine}(\theta)$  points shown in Figure 4.



**If your result was 0.988 instead of 0.5,** it means your calculator is set to calculate the sine of an angle on the radian scale instead of the degree scale. Most calculators have rad/deg or radians/degrees setting that you can choose to tell the calculator treat the  $\theta$  value for trig calculations as either radians or degrees.

**Radians vs. Degrees.** The degree scale splits a circle into 360 one-degree segments. The radian scale splits it up into  $2 \times \pi$  segments. So  $2 \times \pi$  radians =  $360^\circ$  and  $\pi$  radians =  $180^\circ$ . To convert a degree measure to radians, multiply by  $\pi/180^\circ$ , and to convert from radians to degrees, multiply by  $180^\circ/\pi$ .

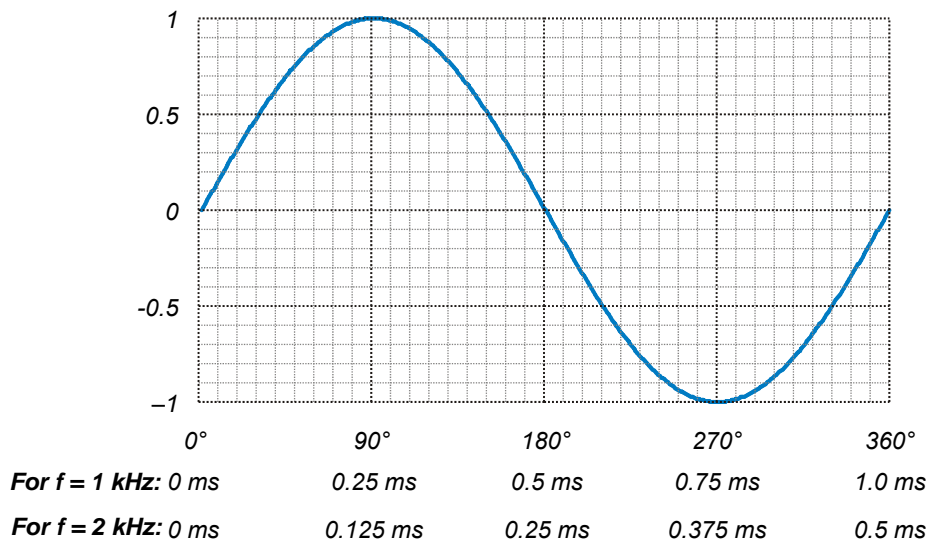
## SINE WAVE PHASE ANGLE AND PHASE SHIFT

Each oscilloscope sine wave display in Chapter 7 is a series of voltage measurements, and each voltage measurement is separated by a small time increment. When the BASIC Stamp or Parallax USB oscilloscope synthesizes a sine wave, it uses increments of time instead of degree angles to calculate the sine wave output voltage. However, there is an important relationship between degrees and time increments in sine waves, and it's called *phase angle*.

The x-axis in our  $\text{sine}(\theta)$  vs  $\theta$  graph is in terms of degrees. In contrast, the x-axis in the PropScope's Oscilloscope screen is in terms of time. Each amount of time corresponds to a certain number of degrees into a cycle of a sine wave. Certain test measurements use degrees instead of time increments to describe how far into a sine wave a point is. These degree measurements are called *phase angle measurements*.

One important fact to keep in mind about phase angle measurements is that the sine wave's frequency dictates the time increment that corresponds to a given angle in the sine wave. Figure 6 shows an example. For a 1 kHz sine wave, a  $90^\circ$  phase angle corresponds to time  $t = 0.25$  ms, but for a 2 kHz sine wave that same  $90^\circ$  angle corresponds to a time  $t = 0.125$  ms instead.

**Figure 6: Sine ( $\theta$ ) Vs.  $\theta$  and Time for 1 and 2 kHz Signals**



You can convert a phase for any frequency to a degree measurement by keeping in mind that the ratio of  $\theta$  to  $360^\circ$  is the same as the ratio of the time ( $t$ ) to the cycle's period ( $T$ ). From that, a phase angle equation is simple. Just multiply both sides of the equation by  $360^\circ$  to solve for  $\theta$ .

$$\frac{t}{T} = \frac{\theta}{360^\circ} \rightarrow \theta = \frac{t}{T} \times 360^\circ$$

The equation  $\theta = (t \div T) \times 360$  is saying that the phase angle  $\theta$  is equal to the time  $t$  of a point on the sine wave, divided by its period  $T$ , and multiplied by  $360^\circ$ . For example, let's say we have a 2 kHz sine wave. That's a frequency, but we need to know its period  $T$ . Remember the period is the reciprocal of frequency, so you can use  $T = 1 \div f$  to calculate the period.

$$T = \frac{1}{f} \rightarrow T = \frac{1}{2000 \text{ Hz}} = 0.0005 \text{ s} = 0.5 \text{ ms}$$

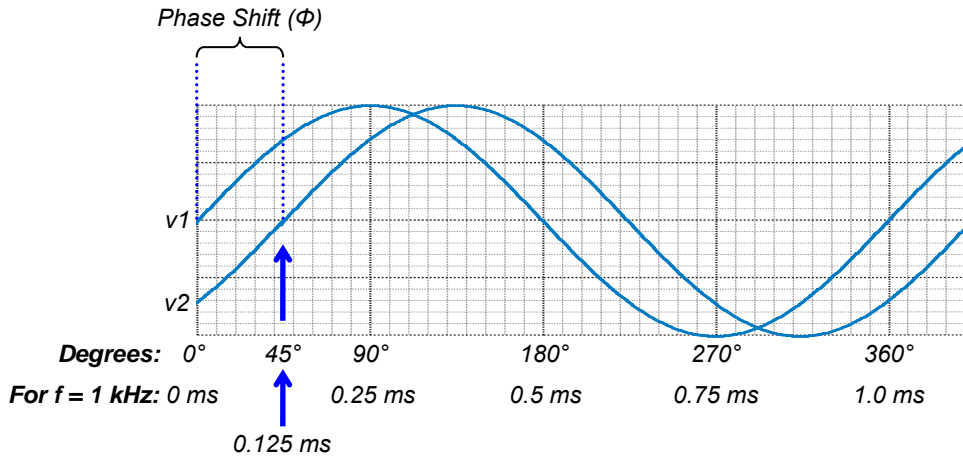
Let's say we want to know the phase angle of a point that's  $t = 0.375$  ms into a cycle of the 2 kHz sine wave. Since we know its period is 0.5 ms, we have all the values we need to calculate the phase angle.

$$\theta = \frac{t}{T} \times 360^\circ = \frac{0.375 \text{ ms}}{0.5 \text{ ms}} \times 360^\circ = 270^\circ$$

- ✓ Take a look at Figure 6. Does the 0.375 ms match with  $270^\circ$  for a 2 kHz signal?
- ✓ Repeat this exercise for 0.125 ms in a 2 kHz signal and 0.25 ms in a 1 kHz signal. Make sure to check your results against Figure 6.
- ✓ Try 0.125 ms for a 1 kHz signal. The correct answer is  $45^\circ$ .

Some circuits introduce delays between input and output signals. For sine waves, this delay can be expressed as a phase angle measurement. For example, all the points in the 1 kHz signal labeled  $v_2$  in Figure 7 occur 0.125 ms later than the corresponding points in the  $v_1$  signal. Using phase angle math, we can calculate that  $v_2$  lags behind  $v_1$  by  $45^\circ$ , or that  $v_1$  leads  $v_2$  by  $45^\circ$ . In oscilloscope terminology, the expressions: " $v_1$  leads  $v_2$ " and " $v_2$  lags  $v_1$ " are common. The angle  $\Phi$  is negative when measured from a reference sine wave to another that lags, or positive when measured to another sine wave that leads. So, the  $\Phi$  between  $v_1$  and  $v_2$  is  $-45^\circ$ , and between  $v_2$  and  $v_1$  is  $+45^\circ$ .

**Figure 7: Identical Sine Waves with Different Phases**



**Phase Angle Measurements:** These measurements can be used to determine properties of circuits and/or the effects they have on signals. In Chapter 7, Activity #6, you will use phase angle tests to determine the delay an RC circuit adds to a sine wave.

**System Stability:** Phase angle measurements are also taken to ensure systems remain stable. An example of a stable system is public address (PA) system where somebody speaks into a microphone, and their voice is amplified and played by a loudspeaker for an audience. In a stable PA system, the amplifier is properly tuned and the speakers are oriented to minimize the sound energy that goes back into the microphone. So the system amplifies the speaker's voice, and not what comes from the loudspeakers.



An example of an unstable system is when the person tries to say something into the PA system's microphone, and all the audience can hear is a loud, high-pitched whine. When the person tried to speak into the microphone, his/her voice was initially amplified by the amplifier and loudspeaker, but the microphone picked up the sound from the loudspeaker, and amplified it too. After that, the system gets stuck in an unstable feedback loop of amplifying what the microphone picks up from the loudspeaker.

The PA system is just one example of a system stability application. Ensuring system stability is part of designing many circuits and systems, including switching power supplies, automated ovens, and motor controllers. System stability has to be incorporated into many mechanical and electromechanical designs as well, including bridges and buildings, automobile cruise control, and aircraft autopilots.

## VOLTAGE SINE WAVE FUNCTION

Figure 8 shows the sine wave function for plotting voltage over time. Variations on this equation are used in place of pictures to describe the behavior of sine waves in many textbooks and measurement practices. The  $v$  term is the voltage output result of the equation, and the  $t$  term is the time input. The other terms represents a properties of the sine wave.  $A$  is the peak amplitude, which is half the peak-to-peak oscilloscope amplitude. The  $f$  term is frequency,  $\Phi$  is phase shift angle, and  $k$  is DC offset.

**Figure 8: Sine Wave Function Returns Voltage for a Given Time**

Voltage (y-axis) Variable                      Time (x-axis) Variable

$$v = A \times \sin \left[ (360^\circ \times f \times t) + \Phi \right] + k$$

Peak Amplitude                      Frequency                      Phase Shift Angle                      DC Offset



### Angular Frequency



The term  $360^\circ \times f$  has an angle multiplied by a frequency, and is called an *angular frequency*. When you multiply an angular frequency by an amount of time, the result is an angle. That's because a sine wave frequency expressed in Hz has seconds in the denominator, and time has seconds in the numerator. When you multiply time by angular frequency, the seconds cancel, and the result is an angle measurement. For example:

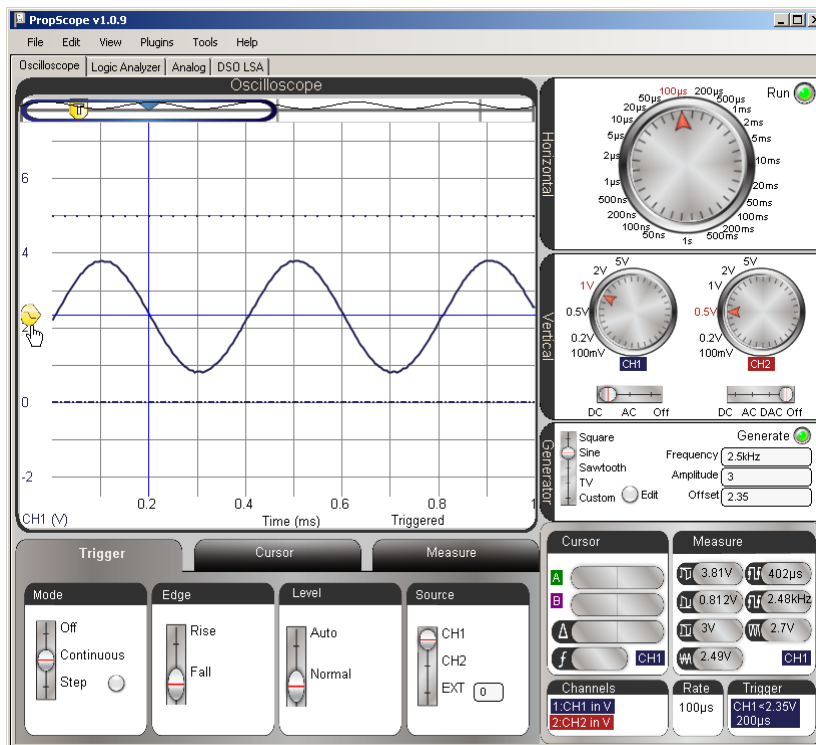
$$\begin{aligned}\theta &= 360^\circ \times 1000 \text{ Hz} \times 0.125 \text{ ms} \\ &= 360^\circ \times \frac{1000}{s} \times 0.000125 \text{ s} = 45^\circ\end{aligned}$$

With the equation in Figure 8, you can plot graphs of any of the voltage sine waves the PropScope function generator creates. For example, you could use  $v = A \times \sin[(360^\circ \times f \times t) + \Phi] + k$  with a graphing calculator, spreadsheet, or calculator and graph paper to plot a graph of the sine wave in Figure 9. This sine wave is the same one in Figure 7-9 from Chapter 7, Activity #2, *Understanding Signals with the PropScope v1.0*.

The  $v$  term in the sine wave equation would be your y-axis variable, and the  $t$  term would be your x-axis variable. You can get the  $A$ ,  $f$ , and  $k$  terms from the Generator panel.  $A$  is the peak amplitude, which is half the peak-to-peak Amplitude in the Generator panel's Amplitude setting. So that's  $A = 3 \text{ V} \div 2 = 1.5 \text{ V}$ . The **f term** is the frequency, which is 2.5 kHz or 2500, and the  $k$  term is the Offset of 2.35 V. If you look carefully, the sine wave's cycle is just starting at 0 ms, so the sine wave has  $0^\circ$  phase shift and  $\Phi$  is  $0^\circ$ . Your equation would look like this:

$$v = 1.5 \text{ V} \times \sin[(360^\circ \times 2500 \text{ Hz} \times t) + 0^\circ] + 2.35$$

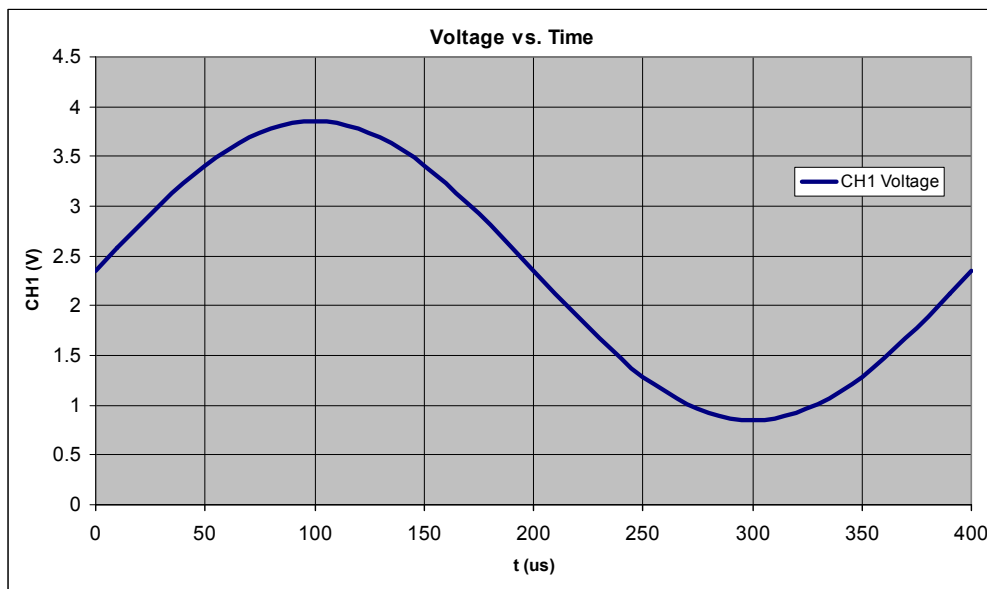
Figure 9: Sine Wave



For graphing purposes, 40 points usually results in a pretty smooth sine wave cycle plot without requiring too much processing. Check the Measure display in Figure 9. It shows that one cycle of the sine wave is  $400\ \mu\text{s}$ , or  $0.4\ \text{ms}$  if you use the time axis at the bottom of the oscilloscope. With 40 points to plot, each time increment should be  $10\ \mu\text{s}$  apart because  $10\ \mu\text{s} \times 40\ \text{points} = 400\ \mu\text{s}$ . For your calculator or spreadsheet,  $10\ \mu\text{s}$  would be  $10 \times 10^{-6}$  or  $0.00001$ .

With  $10\ \mu\text{s}$  increments, the first value of  $t$  in  $v = 1.5V \times \sin[(360^\circ \times 2500\ \text{Hz} \times t) + 0^\circ] + 2.35$  would be  $0\ \text{s}$ . The second point would be  $10 \times 10^{-6}\ \text{s}$ , the second point would be  $10 \times 10^{-6}\ \text{s}$ , the third point would be  $20 \times 10^{-6}\ \text{s}$ , and so on, all the way up through  $390 \times 10^{-6}$ . To give the plot symmetry, you can add  $400 \times 10^{-6}$ , but keep in mind that it's really the beginning of the next cycle. Figure 10 shows an example of one cycle of  $v = 1.5V \times \sin[(360^\circ \times 2500\ \text{Hz} \times t) + 0^\circ] + 2.35$  plotted with a spreadsheet program.

**Figure 10: Spreadsheet Plotted Version of Sine Wave from Figure 9**



### Your Turn: Plot a Sine Wave

- ✓ Set up a  $1\ \text{kHz}$ ,  $4\ \text{V}_{\text{pp}}$  sine wave with  $2\ \text{V}_{\text{DC}}$  offset, no phase delay in the PropScope.
- ✓ Substitute your peak amplitude, frequency, phase delay, and offset terms into  $v = A \times \sin(360^\circ \times f \times t + \Phi) + k$ .
- ✓ Set it up for 40 points of graphing in one cycle.
- ✓ Graph it with the graphing tools of your choice.

### RADIAN MEASUREMENTS

Physics and engineering textbooks tend to rely on radian angle measurements instead of degrees. In the radian measurement system, there are  $\pi$  radians in  $180^\circ$ , which means that there are  $2 \times \pi$  radians in  $360^\circ$ . In the radian system, our sine function would be:

$$v = A \times \sin[(2 \times \pi \times f \times t) + \Phi] + k$$

Like  $360^\circ \times f$ , the term  $2 \times \pi \times f$  is also an angular frequency, but it's in radians per second instead of degrees per second. For a given time value, the seconds in the  $t$  term's numerator cancel with the seconds in the  $f$  term's denominator, and the result is a radian value.

The term used to express angular frequency in physics and engineering textbooks is the Greek letter omega  $\omega$ , where  $\omega = 2 \times \pi \times f$ . Substitute for  $\omega$  for  $2 \times \pi \times f$  in the sine wave equation makes it look like this:

$$v = A \times \sin(\omega \times t + \Phi) + k$$

## DC VOLTAGE VS. AC RMS VOLTAGE

In DC circuits, if a voltage is applied to a resistor, it causes current to pass through it. This in turn causes the resistor to dissipate power by giving off heat. For example, you can feel a 100  $\Omega$  resistor dissipate heat if you apply 5 V to it:

- ✓ Take a 100  $\Omega$  resistor and plug one end into Vss and the other into Vdd.
- ✓ Wait a minute or two.
- ✓ Touch the resistor's ceramic case with your fingertip; it should feel hot to the touch.
- ✓ Unplug the resistor immediately to conserve your battery.

The power the resistor dissipates is the voltage applied across it multiplied by the current passing through it:

$$P = V \times I \quad (\text{DC power equation in terms of } V \text{ and } I)$$

There are a lot of measurements where the voltage and resistance are known, like 5 V across a 100  $\Omega$  resistor. For calculating power, we need V and I instead of V and R. Ohm's Law can be used to solve for I in terms of V and R, and then the result is substituted into the DC power equation's I term. That way, you won't have to manually measure the current each time.

$$V = I \times R \rightarrow I = \frac{V}{R} \quad (\text{Ohm's Law})$$

$$P = V \times I = V \times \frac{V}{R}$$

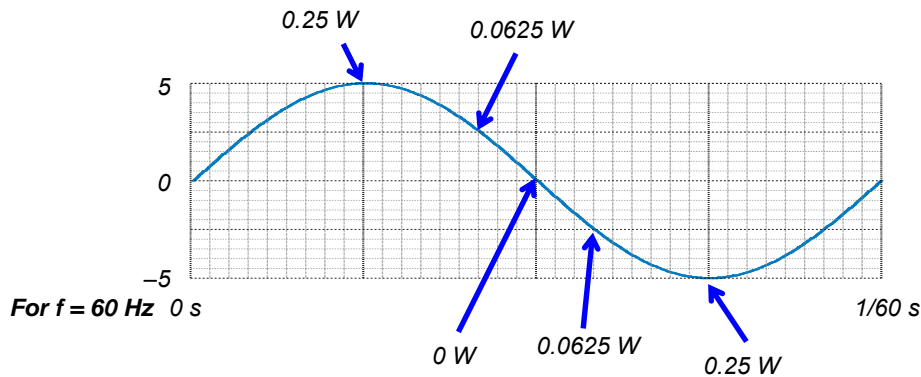
$$\rightarrow P = \frac{V^2}{R} \quad (\text{DC power equation in terms of } V \text{ and } R)$$

Now, we can calculate the power dissipated by the resistor. The power is expressed in watts, which is abbreviated with W, and it's equivalent to units of volt-amps or volts<sup>2</sup>/ohm. Since the resistors in the Understanding Signals kit are rated for 1/4 W, we would not want to increase the voltage to dissipate more power.

$$P = \frac{V^2}{R} = \frac{(5V)^2}{100\Omega} = \frac{25V^2}{100\Omega} \approx 0.25V^2/\Omega = 0.25W$$

How much power would the resistor dissipate if you instead applied a 60 Hz, 5 Vp (10 Vpp) sine wave to the resistor? For one thing, it would dissipate less power than with 5 V<sub>DC</sub>, and Figure 11 shows why. There are only two instants in every cycle when 5 V is applied, and that's +5 V at the top peak, and -5 V at the bottom peak. During the rest of the time, the power is a sequence of instantiations values that result from the way the sine wave's voltage varies. For example, the instantaneous power at 135° and 225° phases is only 0.0625 W. If you are wondering why the power wouldn't be negative when the voltage is negative, the resistor dissipates just as much heat regardless of which direction the current flows. You can also see this in the V<sup>2</sup> term in P = V<sup>2</sup>/R. When you multiply -5 V by -5 V, the result is +25 V, just like it would be if you used +5 V.

**Figure 11: Instantaneous Power Examples with 5 Vpp Applied to a 100 Ω Resistor**



For power calculations, sine wave voltage supplies (commonly called AC supplies) are measured in RMS voltage. RMS stands for *root-mean-square*, and it is a calculation of the voltage that would result in the same amount of power dissipated by a resistor as a DC voltage. The RMS voltage of a sine wave is the peak voltage divided by the square root of 2.

$$V_{RMS} = \frac{V_P}{\sqrt{2}}$$

So, the amount of power a 5 Vp sine wave would supply to a 100 Ω resistor is:

$$P = \frac{V_{RMS}^2}{R} = \frac{\left(\frac{V_P}{\sqrt{2}}\right)^2}{R} = \frac{12.5}{100} = 0.125W$$

To figure out what voltage should be applied to get the same amount of power out of a sine wave, the first step would be to rearrange the power equation terms to solve for  $V_{RMS}$ . Then, multiply that value by root-2 for peak voltage, or twice that for peak-to-peak voltage.

$$P = \frac{V_{RMS}^2}{R} \rightarrow V_{RMS} = \sqrt{P \times R}$$

$$V_{RMS} = \sqrt{0.125W \times 100\Omega} = 3.54V$$

$$V_P = V_{RMS} \times \sqrt{2} = 3.54V \times 1.414 = 5.00V$$

$$V_{PP} = 2 \times V_P = 2 \times 5.00V = 10.00V$$

So, a function generator would need to deliver an AC supply of 5.00 Vp = 10.00 Vpp to make the resistor dissipate the same amount of power as 5 V<sub>DC</sub>.

All devices you plug into wall outlets dissipate power. In the United States, wall outlet voltage is in the neighborhood of 120  $V_{RMS}$ , and oscillates at 60 Hz. In terms of peak voltage, that would be 170 V<sub>p</sub>, or in terms peak-to-peak voltage, it would be 340 V<sub>pp</sub>.

$$V_{RMS}(US\ Outlet) \approx \frac{170V_P}{\sqrt{2}} \approx 120V_{RMS}$$



**DO NOT TRY TO MEASURE WALL OUTLET VOLTAGE WITH THE PROPSCOPE.**

A stock PropScope cannot be used to measure wall outlet voltage. With the probes set to X10, the PropScope can measure up to 200 V<sub>pp</sub>. Since US wall outlet voltage is in the 340 V<sub>pp</sub> neighborhood, it is well outside this range.

Shocks that result from mistakes while attempting to measure wall outlet voltages can be fatal. If you are interested in these types of measurements, seek qualified training in proper measurement procedures and safety precautions, and use only equipment that has been designed, rated, and certified for the measurements you take.



## Filter Error Propagation Example

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If there are uncertainties about any of your measurements or values, those uncertainties will propagate through all your calculations. For example, if two resistors both have 5% tolerances, it means that we have an uncertainty of 5% about the resistors value. For example, a 10 kΩ resistor with a 5% tolerance could be up to 5% larger or smaller than its *nominal* (named) value. These errors are usually denoted by the lower-case Greek letter delta  $\delta$ . For a resistor error margin, that might be  $\delta R$ , and for our 10 kΩ resistor, the error could be as large as 500 or -500.

$$\delta R_1 = 10,000\Omega \times \pm 0.05 = \pm 500\Omega$$

Let's say that two resistors are connected in series. Their values would be added to determine the total resistance, and the amount of uncertainty about the value would be the square root of the sum of the squares of each uncertainty:

$$\delta R = \sqrt{\delta R_1^2 + \delta R_2^2 + \dots}$$

If we add two resistor values, the maximum error based on the two 5% uncertainties would be:

$$\delta R = \sqrt{(500\Omega)^2 + (500\Omega)^2} \approx 707\Omega$$

So, the result of  $R_1 + R_2$  with uncertainty accounted for would be:

$$R_1 + R_2 = 10\text{ k}\Omega + 10\text{ k}\Omega = 20\text{ k}\Omega \pm 707\Omega$$

A calculation we saw more recently is  $R \times C$ . With our 10 kΩ resistor and 0.01 μF capacitor, the two values multiplied together are:

$$\begin{aligned} RC &= R \times C = (10 \times 10^3) \times (0.01 \times 10^{-6}) \\ &= 1 \times 10^{-4} = 0.0001 \end{aligned}$$

Here is the uncertainty for two values multiplied:

$$\delta RC = \pm(R \times C) \times \sqrt{\left(\frac{\delta R_1}{R_1}\right)^2 + \left(\frac{\delta C}{C}\right)^2}$$

We already know that  $\delta R_1 = 500\Omega$ , and for a 0.01 μF capacitor with 5% tolerance,  $\delta C = \pm 0.05 \times 0.01 \times 10^{-6} = \pm 5 \times 10^{-10} = \pm 0.5\text{ nF}$ . So, now we can calculate the uncertainty of that multiplication.

$$\begin{aligned} \delta RC &= \pm(10 \times 10^3 \times 0.01 \times 10^{-6}) \times \sqrt{\left(\frac{500}{10,000}\right)^2 + \left(\frac{5 \times 10^{-10}}{0.01 \times 10^{-6}}\right)^2} \\ &= \pm 1 \times 10^{-4} \times \sqrt{0.05^2 + 0.05^2} \\ &= \pm 0.0001 \times 0.0707 \\ &= \pm 7.07 \times 10^{-6} \end{aligned}$$

With this in mind, our cutoff frequency is really:

$$f_c = \frac{1}{2\pi(RC \pm \delta RC)}$$

Using the positive and negative values of  $\delta RC$ , we have the endpoints of the range of cutoff frequencies we might expect to see given uncertainties about the actual component values.

$$f_c(\text{max}) \approx \frac{1}{2 \times 3.1416 \times (1 \times 10^{-4} - 7.07 \times 10^{-6})} \approx 1712.6 \text{ Hz}$$

$$f_c(\text{min}) \approx \frac{1}{2 \times 3.1416 \times (1 \times 10^{-4} + 7.07 \times 10^{-6})} \approx 1486.5 \text{ Hz}$$

So, the actual cutoff frequency could fall anywhere in the 1486.5 Hz to 1712.6 Hz range.



## Voltage Divider Percent Error Example

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The difference between predicted and measured values can be quantified in terms of percent error.

$$\%error = \frac{measured - predicted}{predicted} \times 100\%$$

For example, if you expect a voltage to be 2.5 V, but your measurement is 2.3 V, the percent error is:

$$\begin{aligned} \%error &= \frac{2.3V - 2.5V}{2.5V} \times 100\% \\ &= -0.08 \times 100\% \\ &= -8\% \end{aligned}$$

### Resistor Tolerance

*Resistor tolerance* is the percentage that a resistor's actual value can vary from its labeled value. The gold bands on the resistors in your kit indicate that they have +/- 5% tolerances, meaning that their actual values might be 5% larger or smaller than the value the color bar code indicates.

In the case of our brown-black-red color code, that would be 1 k $\Omega$  +/- 5%. Since 5% of 1000  $\Omega$  is 50  $\Omega$ , the actual resistor values could be anywhere between 950  $\Omega$  and 1050  $\Omega$ . The largest error above the expected value would be if R1 = 1050  $\Omega$  and R2 = 950  $\Omega$ , which results in a voltage divider calculation of 5.625 V, which is 5 % higher than 2.5 V. If you try this with resistor values that aren't equal to each other, you might notice slightly different percent errors. That's because of the way the errors carry through the equation, it's called *error propagation*.

### How much error would still be normal in my voltage divider measurement?

Although we expect a measurement of 2.5 V, the actual measurement could be anywhere in the 2.3 to 2.7 V range. The three major sources of error in the circuit and system are:

- Resistor tolerances: +/- 5%
- PropScope voltage tolerance: Although not published at the time of this writing, let's say it's +/- 2%
- Vdd supply voltage tolerance: under +/- 1%

That's a total of about +/- 8%, so the maximum and minimum actual measurements could be:

$$\begin{aligned} V_{max} &= 2.5 V \times 1.08 = 2.7 V \\ &\text{and} \\ V_{min} &= 2.5 V \times 0.92 = 2.3 V \end{aligned}$$

Since that's either 0.2 V above or below 2.5 V, a convenient shorthand for the measurement tolerance is 2.5 V +/- 0.2 V.

## **Errata for Understanding Signals with the PropScope**

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No technical errors reported as of this document's publication date (see footer).

## **Supplement Revision History**

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Original version.